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# CALCULATION OF THE FLOW RATE OF THE HYDRAULIC MIXTURE OF THE HYDRAULIC TRANSPORT COMPLEX BY ANALYTICAL METHOD

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**Abstract.** The article proposes a multiplicative formula for calculating the analytical flow rate of a hydraulic transport complex, taking into account the parameters of the pipeline and centrifugal pumps used, as well as the characteristics of the polydisperse solid material being transported. The formula involves the use of a power law instead of a logarithmic law to describe the dependence of the hydraulic resistance coefficient on the Reynolds number, and the approximation of the flow-pressure characteristics of centrifugal pumps by a special power function instead of the traditional quadratic binomial. The multiplicity of the proposed formula lies in the fact that the flow rate of the hydraulic mixture of the hydraulic transport complex is determined by the product of three factors, one dimensional and two dimensionless, each of which takes into account the influence of a separate group of factors. The characteristic flow rate is a single dimensional factor and takes into account the influence of the pipeline diameter and the kinematic viscosity coefficient of the hydraulic mixture. The polydispersity parameter of the transported material is a dimensionless quantity that takes into account the influence of the geodetic slope of the pipeline and the parameters of fine fractions, i.e., particles with a size of 0.1 mm to 2 mm, and lump fractions, i.e., particles with a size greater than 2 mm. The pump parameter, is also a dimensionless quantity and is determined by the concentration of the hydraulic mixture, the properties of the solid phase and its fine fractions, i.e., particles with a size of less than 0.1 mm, as well as the approximation coefficients of the flow-pressure characteristics of the installed centrifugal pumps. The formula allows one to study the dependence of the flow rate of the hydraulic mixture of the hydraulic transport complex on various values by analytical methods, for example, to determine the extrema of the function by comparing its derivatives with zero, or to determine the permissible intervals of change of the parameters included in the formula by limiting the flow rate of the hydraulic mixture.

**Keywords:** pressure hydraulic transport, flow-pressure characteristic, centrifugal pump, hydraulic mixture, flow rate.

## 1. Introduction

One of the promising areas of industrial activity in Ukraine is the extraction of minerals that are necessary for domestic consumption and have the potential to be sold on the world market [1–4]. Ukraine's potential for rare earth minerals and some other minerals is considered by world experts to be very promising. Domestic experts view this potential as one of the components of the country's economic and financial security, as well as a guarantee of successful and sustainable development of a number of regions [5, 6]. If we consider only the economic and technological factors of profitability and competitiveness of domestic mining enterprises in the context of global competition with foreign enterprises, most experts point to the high energy and resource intensity of domestic technologies [4, 7]. Most of the technological schemes and methods for the extraction, transportation, and processing of minerals were created back in the days of the former USSR, when electricity was cheap and water and land resources could be used without restraint. This situation gradually changed after Ukraine gained independence, but most technological solutions either remained unchanged, were adapted, or underwent minimal modernization. Recent years, which have been marked by significant changes in the energy sector not only in Ukraine but also in most European Union countries, have posed a significant challenge for domestic mining companies. Not only have electricity prices changed, but the

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possibility and volume of its use have also decreased. The same can be said about the main natural resource required by all mining enterprises – technical water [8].

Such rapid changes in economic and environmental factors have significantly altered the possibilities for using certain types of industrial transport traditionally used to move ore from the place of extraction to the place of processing [1, 9–11]. Pressure hydrotransport of hydro-mixtures has proven to be the most vulnerable, as it consumes both of the above-mentioned resources – electricity and water. Under such conditions, instead of extending pipelines and moving pumping stations beyond the working face, some domestic enterprises are reducing the length of the main lines of hydraulic transport complexes and organizing the delivery of tailings from the extraction site to stationary pumping stations by road transport.

Thus, the further use of pressure hydraulic transportation technologies requires the improvement of methods for calculating parameters and operating modes, as well as the creation of methods for determining parameters that ensure minimum energy consumption and water consumption in each specific case.

In the 20th century, a new opportunity to reduce the hydraulic resistance of pipes appeared in the practice of pipeline transportation of gas and liquids, namely, the use of polyethylene pipes [12–17]. In the 21st century, this experience began to be implemented in domestic mining enterprises for hydropower complexes that transport hydraulic mixtures [18–21]. However, none of the 18 methods for calculating hydrotranport parameters developed during the Soviet era for steel pipes guarantees a certain accuracy in determining parameters, which leads to increased consumption of electricity and water [1, 2, 22–25].

Thus, the aim of the study is to establish an analytical dependence for calculating the flow rate of the hydraulic mixture of the hydraulic transport complex, taking into account the geometric and physical parameters of the pipeline, the centrifugal pumps used, as well as the characteristics of the polydisperse solid material being transported, and would allow to investigate the dependence of the flow rate of the hydraulic mixture of the hydraulic transport complex on various values by analytical methods, for example, to determine the extrema of the function by comparing its derivatives with zero, or to determine the permissible intervals of change of the parameters included in the formula by limiting the flow rate of the hydraulic mixture.

## 2. Methods

Traditionally, the consumption of hydraulic fluid during the operation of a hydraulic transport complex is determined by the ratio of the flow and pressure characteristics of the pumps and the main line [1, 2, 22–25]. The flow-pressure characteristics of centrifugal pumps are usually approximated by a quadratic or linear polynomial [1, 2, 9–11]. That is, there is always a free term and linear or quadratic terms in the equation. The flow-pressure characteristic of the pipeline during the flow of a hydro mixture consists of three terms [1, 2, 9–11]: a term caused by the geodetic height difference and the presence of lumpy fractions in the solid phase, i.e., particles larger than 2 mm; a term caused by the presence of fine fractions in the solid phase, i.e., particles with a size of 0.1 mm to 2 mm; and a component caused by the hydraulic resistance of the liquid phase of the hydro mixture and the presence of fine fractions in the solid phase, i.e., particles with a size of less than 0.1 mm.

The component caused by the geodetic height difference and the presence of lumpy fractions in the solid phase does not depend on the flow rate of the hydraulic mixture and therefore is included in the free term of the equation. The component caused by the presence of fine fractions in the solid phase depends on the flow rate of the hydraulic mixture inversely proportionally. This creates the prerequisites for the final equation for determining the flow rate to be at least of the third degree. The component caused by the hydraulic resistance of the liquid phase of the hydraulic mixture and the presence of fine fractions in the solid phase depends on the flow rate of the hydraulic mixture in a nonlinear and complex manner. For steel pipes, this dependence is traditionally considered to be directly proportional to the square of the flow rate, but the coefficient of proportionality, the so-called hydraulic friction resistance coefficient, depends on the flow rate included in the Reynolds criterion, according to the logarithmic law [1, 2, 9–11].

As a result, with the traditional approach, the equation for determining the flow rate will be a quadratic equation if water is transported, or a cubic equation if a hydro mixture is transported, one of the coefficients of which contains a nonlinear component that is inversely proportional to the logarithm of the flow rate. None of the analytical methods can solve such equations.

Studies of the hydraulic resistance of steel pipes during the hydrotransport of hydro-mixtures, conducted by domestic scientists in the second half of the 20th century, indicate the possibility of using a non-logarithmic, but a power law [9, 18–20]. At the beginning of the 21st century, foreign experts on the use of polyethylene pipes for water supply recommended a similar law for hydraulic calculations of pipes made of polymer materials [18–20]. This, firstly, allows generalizing the calculation of the hydraulic slope when water flows through pipes made of different materials, and, secondly, allows eliminating the logarithm in the equation for determining the flow rate.

The results of research conducted by specialists at the M.S. Poliakov Institute of Geotechnical Mechanics of the National Academy of Sciences of Ukraine indicate that the use of the power law to determine the hydraulic gradient allows the equation for determining the flow rate to be reduced to a polynomial with fractional powers. However, this equation also has no analytical solution, since it is impossible to reduce it to a third- or second-degree equation by substitution [3, 4, 18–20]. This obstacle disappears if, to calculate the component of the hydraulic gradient of the hydraulic mixture, which is due to the presence of small fractions in the solid phase, we use the results of our own research, which indicate an inversely proportional dependence of this value on the square root of the hydraulic gradient of water [18, 20]. This approach, with the simultaneous approximation of the flow-pressure characteristics of the pump-suction two-term with a fractional degree, allows us to propose the following equation for calculating the flow rate of the hydraulic mixture during the operation of the hydraulic transport complex:

$$mH_f - mBQ^q = k_z L\alpha\mu Q^{2-p} + \frac{\beta k_z L}{\sqrt{\mu}Q^{\frac{2-p}{2}}} + \gamma k_z L + \alpha\Delta Z\,; \tag{1}$$

$$\mu = \frac{2^{3-2p}M}{\pi^{2-p}}\frac{\nu^p}{gD^{5-p}}\,;\ \alpha = 1 + ASR_1\,;\ \beta = \frac{k_w w}{\sqrt{gd}}ASR_2\,;\ \gamma = fASR_3\,;$$

$$A = \frac{1-SR_1}{1+ArSR_1}Ar\,,$$

where $H_f$ – fictitious pump pressure when the flow rate is zero, m; $B$ – pressure reduction coefficient; $m$ – number of pumps providing transportation; $q$ – the degree of the polynomial used to approximate the head-flow characteristic of the pump; $L$ – track length, m; $\Delta Z$ – difference in geodetic heights between the end and the beginning of the route, m; $Q$ – water consumption entering the main pipeline, $m^3/s$; $k_z$ – local hydraulic resistance coefficient; $p$ – index of the degree in the power law of dependence of the hydraulic resistance coefficient on the Reynolds number, $0 < p < 2$; $D$ – pipeline diameter, m; $M$– coefficient in the power law of dependence of the hydraulic resistance coefficient on the Reynolds number ; $\mu$ – coefficient; $\alpha$ – parameter that takes into account the influence of fine fractions, i.e. particles smaller than 0.1 mm, on the hydraulic slope of the hydro mixture; $\beta$ – parameter that takes into account the influence of fine fractions, i.e. particles ranging in size from 0.1 mm to 2 mm, on the hydraulic slope of the hydraulic mixture; $\gamma$ – parameter that takes into account the influence of coarse fractions, i.e. particles larger than 2 mm, on the hydraulic slope of the hydraulic mixture; $A$ – refined parameter of Archimedes for solid fraction particles; $S$ – volume concentration of the hydro mixture; $R_1$ – weight fraction of fine fractions in the particle size distribution of the solid phase of a hydro mixture; $k_w$ – empirical coefficient, determined according to recommendations; $R_2$ – weight fraction of fine fractions in the granulometric composition of the solid phase of the hydro mixture; $w$ – generalized hydraulic particle size of fine fractions, m/s; $d$ – generalized particle size of fine fractions, m; $f$ – generalized coefficient of friction of particles of lump fractions against the inner surface of the pipeline; $Ar$ – Archimedes parameter of the solid phase of a hydro mixture; $R_3$ – weight fraction of lump fractions in the granulometric composition of the solid phase of the hydro mixture, $g$ - free-fall acceleration, $m/s^2$.

## 3. Theoretical part

Equation (1) is transformed into a dimensionless equation of general form:

$$\alpha\Theta^{3\frac{2-p}{2}} + b\Theta^{\frac{2q+2-p}{2}} - c\Theta^{\frac{2-p}{2}} + \beta = 0\,; \tag{2}$$

$$\Theta = \frac{Q}{Q_\mu};\ Q_\mu = \pi^{2-p}\sqrt{\frac{gD^{5-p}}{2^{3-2p}M\nu^p}};\ c = i_h - \gamma - \alpha i_g;\ b = \frac{mBQ_\mu^q}{k_z L};$$

$$i_h = \frac{mH_f}{k_z L},\ i_g = \frac{\Delta Z}{k_z L},$$

where $\Theta$ – dimensionless flow rate of hydraulic mixture; $Q_\mu$ – characteristic flow rate, m$^3$/s; $c$ – effective head; $i_h$ – head pressure; $i_g$ – geodetic slope; $b$ – dimensionless specific pressure drop.

The results of the analysis indicate that the last equation has an analytical solution when

$$q = \frac{2-p}{2} \tag{3}$$

or

$$q = 2-p. \tag{4}$$

If (3) holds, then after appropriate transformations, equation (2) is transformed into the following complete cubic equation:

$$x^3 + \frac{b}{\alpha}x^2 - \frac{c}{\alpha}x + \frac{\beta}{\alpha} = 0;\qquad x = \Theta^{\frac{2-p}{2}}, \tag{5}$$

which, after replacement

$$y = x - \frac{b}{3\alpha},$$

is transformed into a cubic equation of canonical form

$$y^3 + py + q = 0;\ p = -\frac{1}{3}\left(\frac{b}{\alpha}\right)^2 - \frac{c}{\alpha};\ q = 2\left(\frac{b}{3\alpha}\right)^3 + \frac{b}{3\alpha}\frac{c}{\alpha} + \frac{\beta}{\alpha}. \tag{6}$$

The last equation by substitution

$$z = \frac{1}{\sqrt[3]{q}}y,$$

after appropriate transformations, leads to the following equation

$$z^3 - \vartheta z + 1 = 0;\qquad \vartheta = \frac{p}{\sqrt[3]{q^2}},$$

whose valid and positive solutions exist when

$$\vartheta \geq \frac{3}{\sqrt[3]{4}}, \tag{7}$$

and are calculated using the following formulas

$$z_1 = 2\sqrt{\frac{\vartheta}{3}}\cos\left(\frac{\tilde{\alpha}}{3}\right);\ z_2 = \sqrt{\frac{\vartheta}{3}}\left(\sqrt{3}\sin\left(\frac{\tilde{\alpha}}{3}\right) - \cos\left(\frac{\tilde{\alpha}}{3}\right)\right);\ \cos\tilde{\alpha} = -\sqrt{\frac{27}{4\vartheta^3}}, \tag{8}$$

where $\vartheta$ – equation parameter.

Of the two valid and positive roots of the equation under consideration, one describes the case of increased flow with increasing pipeline length, and the other, which is approximated by a power dependence on the parameter $\vartheta$:

$$z = 0.669\vartheta^{\frac{3}{5}}, \tag{9}$$

reduction in flow rate with an increase in pipeline length.

Taking into account the proposed approximation, the formula for calculating the flow rate of the hydraulic mixture supplied by the hydraulic transport complex can be written as follows:

$$Q = Q_\mu \left[ \frac{b}{3\alpha} + \frac{0.696\left(3\left(\frac{b}{3\alpha}\right)^2 + \frac{c}{\alpha}\right)^{\frac{3}{5}}}{\left(2\left(\frac{b}{3\alpha}\right)^3 + \frac{b}{3\alpha}\frac{c}{\alpha} + \frac{\beta}{\alpha}\right)^{\frac{1}{15}}} \right]^{\frac{2}{2-p}}. \tag{10}$$

For convenience of further analysis, it is reasonable to rewrite formula (10) in the following form:

$$Q = Q_\mu K,\ K = \left[ 1 + 1.285\left( \frac{\left(1 + 9\frac{c\alpha}{b^2}\right)^9}{1 + 9\frac{c\alpha}{2b^2} + \frac{27\beta\alpha^2}{2b^3}} \right)^{\frac{1}{15}} \right]^{\frac{2}{2-p}} \left(\frac{b}{9\alpha}\right)^{\frac{2}{2-p}}. \tag{11}$$

If (4) holds, then after the appropriate transformations, equation (2) is transformed into the following complete cubic equation of canonical form:

$$z^3 - \vartheta z + 1 = 0,\; z = \sqrt[3]{\frac{\alpha + b}{\beta}\Theta^{\frac{2-p}{2}}};\; \vartheta = \frac{c}{\beta^{\frac{2}{3}}}\frac{1}{\sqrt[3]{\alpha + b}}, \tag{12}$$

where $\vartheta$ – equation parameter.

According to formulas (7) and (8), real and positive solutions of equation (12) exist when

$$\frac{c}{\sqrt[3]{\alpha + b}\beta^{\frac{2}{3}}} \geq \frac{3}{\sqrt[3]{4}},$$

and are calculated using the following formulas

$$Q = Q_{\mu} K, \tag{13}$$

$$K = \begin{cases} \left(\frac{2}{\sqrt{3}}\frac{\sqrt{c}}{\alpha + b}\right)^{\frac{2}{(2-p)}} \cos^{\frac{2}{(2-p)}}\left(\frac{\tilde{\alpha}}{3}\right) \\ \left(\frac{2}{\sqrt{3}}\frac{\sqrt{c}}{\alpha + b}\right)^{\frac{2}{(2-p)}} \left(\frac{\sqrt{3}}{2}\sin\left(\frac{\tilde{\alpha}}{3}\right) - \frac{1}{2}\cos\left(\frac{\tilde{\alpha}}{3}\right)\right)^{\frac{2}{(2-p)}} \end{cases},\; \cos\tilde{\alpha} = -\sqrt{\frac{27(\alpha + b)\beta^2}{4c^3}}.$$

The equation obtained has real and positive solutions only in the case when condition (7) holds, and are calculated by the following formulas (8). Moreover, of the two real and positive roots of the equation under consideration, one, formula (13), describes the case of an increase in flow rate with an increase in the length of the pipeline, and the second, which is approximated by the power dependence (9) from the parameter $\vartheta$ a decrease in flow rate with an increase in the length of the pipeline.

## 4. Results and Discussion

Dependence (11), taking into account the formulas of the quantities included in it, allows us to propose the following multiplicative formula for calculating the consumption of the hydraulic mixture of the hydraulic transport complex:

$$Q = Q_{\mu} Q_d Q_a, \tag{14}$$

$$Q_d = \frac{\left(i_g - fS_0 R_3\right)^{\frac{6}{5(2-p))}}}{1.44^{\frac{2}{2-p}}\left(\frac{k_w w}{\sqrt{2gdS_0}}\right)^{\frac{2}{15(2-p))}}}; \; Q_a = \left(\frac{(s-s_1)^9 (s+s_2)^9}{s(1-s)(s-s_3)^8 (s+s_4)^8}\right)^{\frac{2}{15(2-p)}};$$

$$s_1 = I\left[\sqrt{1+\frac{1}{uI^2}}+1\right]; \; s_2 = I\left[\sqrt{1+\frac{1}{1I^2}}-1\right]; \; I = \frac{1}{2}\frac{i_h - fS_0 R_3}{2i_g - fS_0 R_3}; \; S_0 = \frac{1}{R_1};$$

$$u = ArS_0,$$

$$s_4 = \left(1+\frac{b}{2}\right)\left[\sqrt{1+\frac{1+b}{Ar\left(1+\frac{b}{2}\right)^2}}-1\right]; \; s_3 = \left(1+\frac{b}{2}\right)\left[\sqrt{1+\frac{1+b}{Ar\left(1+\frac{b}{2}\right)^2}}+1\right]; \; s = \frac{S}{S_0},$$

where $Q_d$ – parameter of polydispersity of the transported material; $Q_a$ – pump parameter set; $s$ – relative concentration of the hydro mixture; $s_1$ – the first characteristic value of the relative concentration, determined by the pump pressure when the flow rate is zero; $s_2$ – the second characteristic value of the relative concentration, which is determined by the pump pressure when the flow rate is zero; $I$ – adjusted fictitious pump head when flow rate is zero; $u$ – parameter of the solid phase of a suspension-carrying suspension; $s_3$ – the first characteristic value of the relative concentration, determined by the pump pressure reduction coefficient; $s_4$ – the second characteristic value of the relative concentration, which is determined by the pump pressure reduction coefficient; $S_0$ – hydraulic mixture concentration module.

The characteristic flow rate is the only dimensional factor on the right side of formula (14). To analyze the range of its variation, it is reasonable to write the formula for calculating this value in the following form:

$$Q_\mu = \delta D^k; \; \delta = \sqrt[2-p]{\frac{\pi^{2-p} g}{2^{3-2p} M\nu^p}}; \; k = \frac{5-p}{2-p},$$

which allows us to evaluate the intervals of change in the components of this dependence (Figures 1, 2) and the characteristic flow rate itself (Figure 3).

From Figure 3, it can be concluded that the dependence of the characteristic flow rate on the pipe diameter for all values of the exponent in the power law of the dependence of the hydraulic resistance coefficient on the Reynolds number is increasing, without extrema, and asymptotically approaches certain stable values. At the same time, higher values of the $p$ index correspond to higher values of the characteristic flow rate.

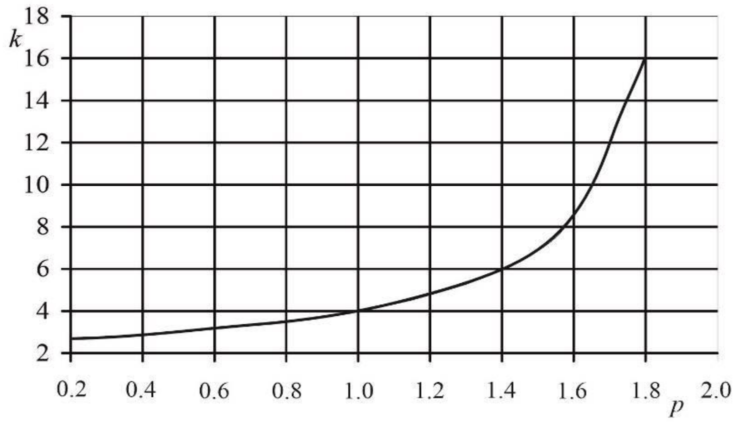


Figure 1 – Change in the degree indicator depending on the characteristic flow rate from the diameter of the pipeline when changing the parameter $p$

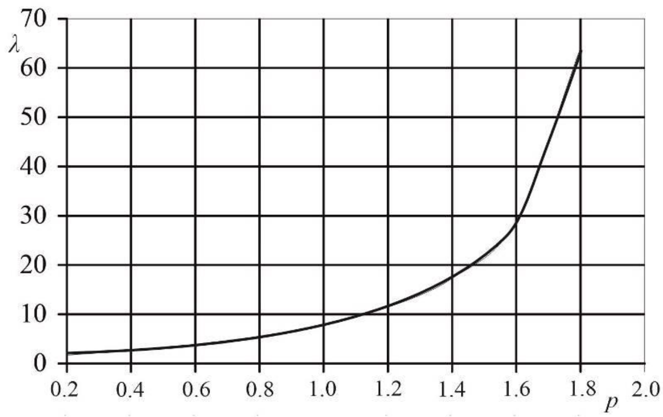


Figure 2 – Change in the logarithm of the coefficient $\lambda = \lg \delta$ depending on the characteristic flow rate from the pipeline diameter when changing the parameter $p$

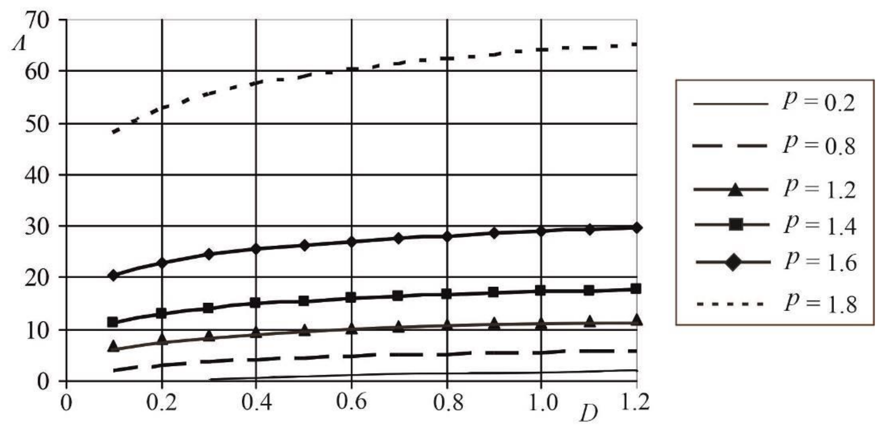


Figure 3 – Dependence $\Lambda = lg\, Q_{\mu}$ of the logarithm of the characteristic flow rate on the pipe diameter for different values of the exponent in the power law of the dependence of the hydraulic resistance coefficient on the Reynolds number

The polydispersity parameter of the transported material is a dimensionless quantity, which can also be rationally rewritten as a power dependence on the main factors, such as the Froude criterion and the fictitious geodetic slope:

$$Q_d = \frac{\delta}{W^n};\ \delta = \frac{\left(i_g - fS_0R_3\right)^{\frac{6}{5(2-p)}}}{1.44^{\frac{2}{2-p}}};\ W = \frac{k_w w}{\sqrt{2gdS_0}};\ n = \frac{2}{15(2-p)},$$

which allows us to estimate the intervals of change in the components of this dependence (Figure 4) and directly the parameter of polydispersity of the material (Figure 5).

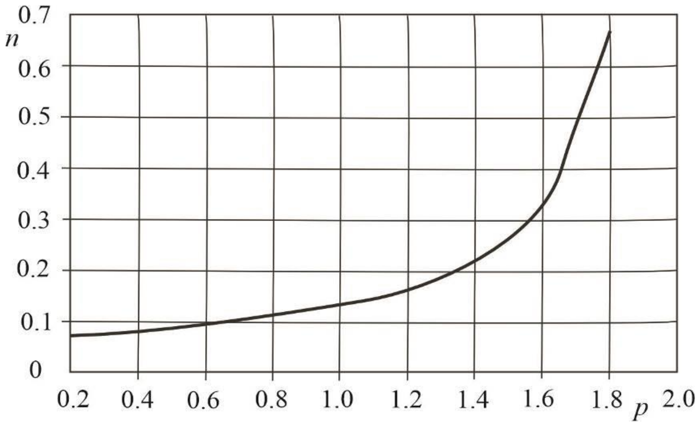


Figure 4 – Change in the degree index depending on the polydispersity parameter of the transported material from the value $p$

From Figure 5, it can be concluded that for all values of the fictitious geodetic slope and the exponent in the power law of the dependence of the hydraulic resistance coefficient on the Reynolds number, the value of the polydispersity parameter of the transported material decreases with an increase in the Froude criterion. In all cases, this dependence has no extrema, and the rate of decrease increases with an increase in the exponent $p$. At the same time, higher values of the fictitious geodetic slope always correspond to higher values of the polydispersity parameter of the transported material.

The last factor on the right side of formula (14) is a dimensionless parameter of the pumps installed on the hydraulic transport complex. This value depends in a complex way on the relative concentration of the hydraulic mixture, which varies from zero to one. Both of these points, zero and one, are special for the pump parameter, since at these points its value becomes infinite. The dependence of this parameter on the concentration of the hydraulic mixture is determined by the values of four more parameters: two characteristic values of the relative concentration, determined by the pump head when the flow rate is zero (Figures 6, 7), and two characteristic values of the relative concentration, determined by the pump head reduction coefficient (Figures 8, 9).

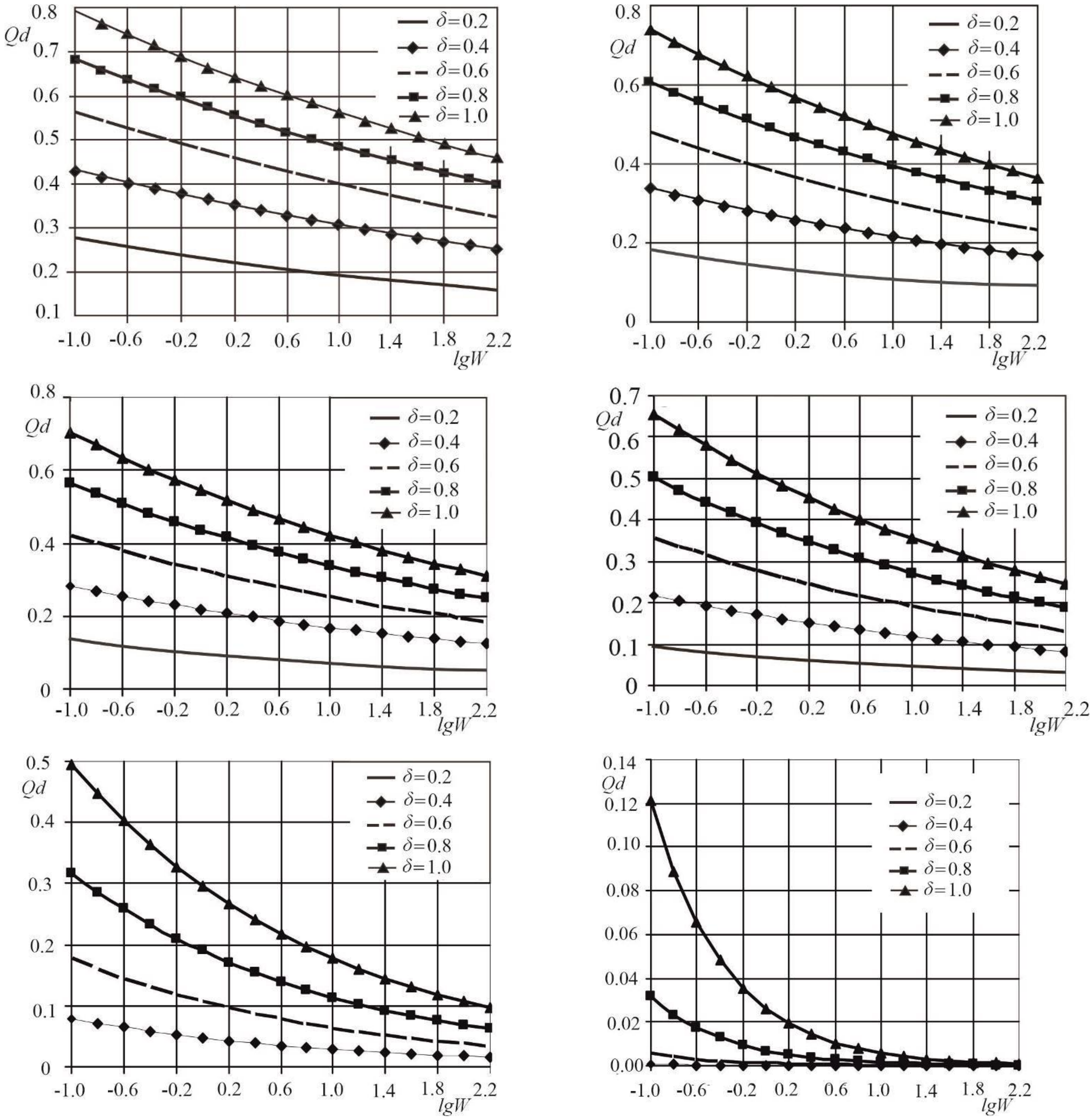


Figure 5 – Dependence of the polydispersity parameter of the transported material on the log-rhyme of the Froude criterion for different values of the fictitious geodetic slope at *p*=0.2; *p*=0.6; *p*=0.8; *p*=1.0; *p*=1.4; *p*=1.8

The factors containing characteristic values of relative concentration determined by pump pressure when the flow rate is zero are located in the numerator of the fraction, while those containing characteristic values of relative concentration determined by the pump pressure reduction coefficient are located in its denominator. Thus, the first characteristic value of relative concentration, which is determined by the pump pressure reduction coefficient $s_3$ (Figure 8), is also a special point for the pump parameter, since here its value also becomes infinite. At the same time, the value of the first characteristic value of the relative concentration, which is determined by the pump pressure when the flow rate is zero, $s_1$ (Figure 6), limits the concentration of the hydraulic mixture from below, since the dimensionless parameter of the installed pumps must be positive.

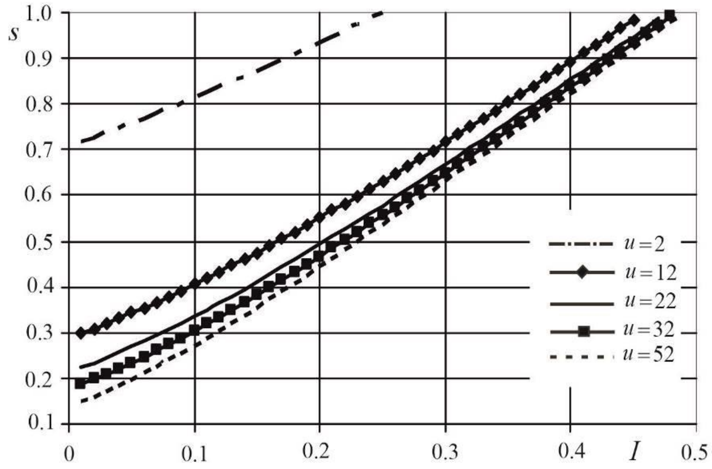


Figure 6 – Dependence of the first characteristic value of the relative concentration, determined by the pump head when the flow rate is zero, on the corrected fictitious pump head when the flow rate is zero, for different values of the solid phase parameter of the suspended suspension

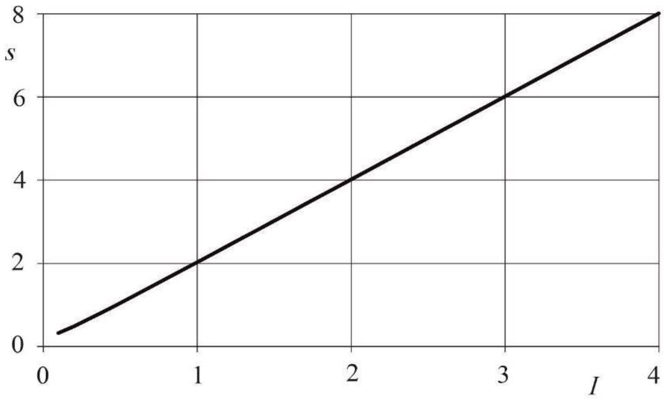


Figure 7 – Dependence of the second characteristic value of the relative concentration, determined by the pump head when the flow rate is zero, on the corrected fictitious pump head when the flow rate is zero

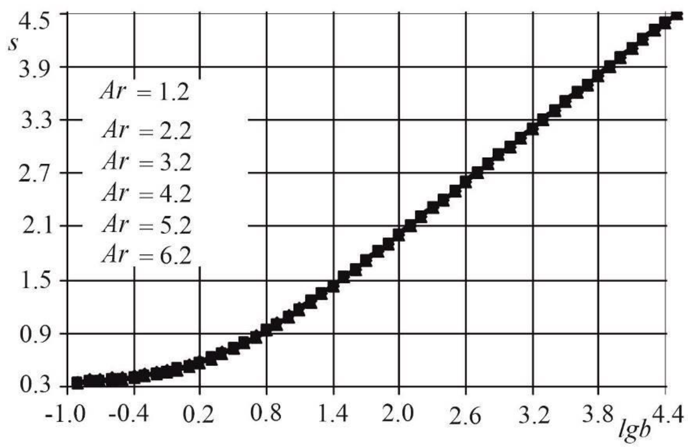


Figure 8 – Dependence of the first characteristic value of the relative concentration, determined by the pump pressure reduction coefficient, on the logarithm of the dimensionless specific pressure reduction for different values of the solid phase density

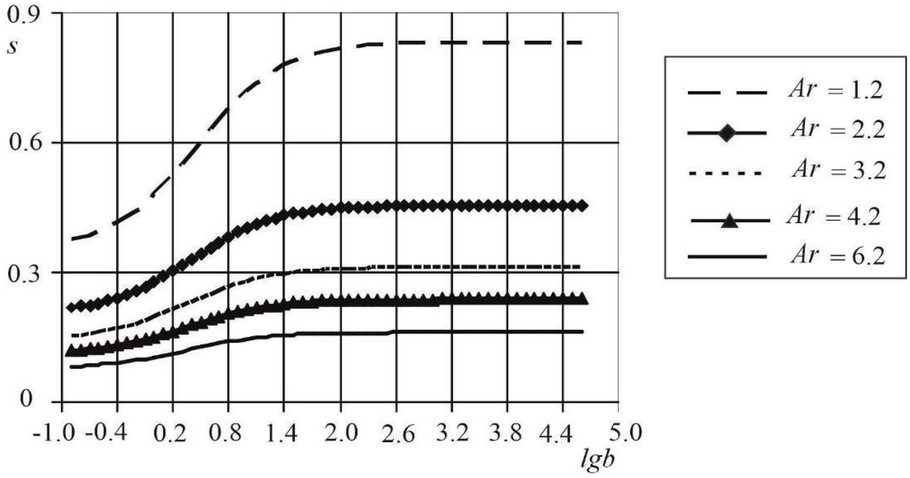


Figure 9 – Dependence of the second characteristic value of relative concentration, determined by the pump pressure reduction coefficient, on the logarithm of the dimensionless specific pressure reduction for different values of solid phase density

As can be seen from Figure 7, the second characteristic value of the relative concentration, determined by the pump head when the flow rate is zero, is almost independent of the value of the solid phase parameter of the suspension and is determined exclusively by the value of the corrected fictitious pump head when the flow rate is zero. A similar conclusion can be drawn from Figure 8 regarding the first characteristic value of the relative concentration, which is determined by the pump pressure reduction coefficient. This value is almost independent of the density of the solid phase and is determined exclusively by the value of the dimensionless specific pressure reduction (Figure 8). Unlike these two parameters, the value of the second characteristic value of the relative concentration, determined by the pump pressure reduction coefficient, as follows from Figure 9, significantly depends on the value of the solid phase density, as does the value of the first characteristic value of the relative concentration, which is determined by the pump head when the flow rate is zero, as can be seen in Figure 6.

Figures 6–9 show that all characteristic values of relative concentration increase with an increase in the main factors determining their values, in the first case – the corrected fictitious pump head when the flow rate is zero, in the second – the dimensionless specific head reduction. For all these dependencies, higher values of the parameters characterizing the properties of the solid phase of the hydro mixture correspond to lower values of the characteristic values of the relative concentration.

**5. Conclusions**

Thus, the article proposes a multiplicative formula for calculating the flow rate of the hydraulic mixture of the hydraulic transport complex in analytical form, taking into account the parameters of the pipeline and centrifugal pumps used, as well as the characteristics of the polydisperse solid material being transported. The formula involves the use of a power law instead of a logarithmic law to describe the dependence of the hydraulic resistance coefficient on the Reynolds number, and the approximation of the flow-pressure characteristics of centrifugal pumps by a special power function instead of the traditional quadratic binomial.

The multiplicity of the proposed formula lies in the fact that the flow rate of the hydraulic mixture of the hydraulic transport complex is determined by the product of three factors, one dimensional and two dimensionless, each of which takes into account the influence of a separate group of factors. The characteristic flow rate is the only dimensional factor and takes into account the influence of the pipeline diameter and the kinematic viscosity coefficient of the hydraulic mixture. The polydispersity parameter of the transported material is a dimensionless quantity that takes into account the influence of the geodetic slope of the pipeline and the parameters of fine fractions, i.e., particles with a size of 0.1 mm to 2 mm, and lump fractions, i.e., particles with a size greater than 2 mm. The parameter of the installed pumps is also a dimensionless quantity and is determined by the concentration of the hydraulic mixture, the properties of the solid phase and its fine fractions, i.e., particles with a size of less than 0.1 mm, as well as the approximation coefficients of the flow-pressure characteristics of the installed centrifugal pumps.

The formula allows us to study the dependence of the flow rate of the hydraulic mixture of the hydraulic transport complex on various values using analytical methods, for example, to determine the extrema of the function by comparing its derivatives with zero, or to determine the permissible intervals of change in the parameters included in the formula by limiting the flow rate of the hydraulic mixture.

A significant drawback of the proposed formula is the dependence of all factors on the weight fraction of fine fractions in the granulometric composition of the solid phase of the hydraulic mixture and on the exponent in the power law of the dependence of the hydraulic resistance coefficient on the Reynolds number. Since these characteristics are used in determining all three factors of the multiplicative formula for obtaining the derivative of the flow rate of the hydraulic mixture, they require complex and cumbersome analytical calculations.

It should be noted that the proposed formula for determining the flow rate of the hydro-transport complex's hydro-mixture assumes the presence of all three fractions in the transported material. The possibility of using it in cases of two-fractional or single-fractional materials requires additional justification and research. Since the absence of some fractions, for example, fine fractions, i.e., particles with a size of 0.1 mm to 2 mm, leads to fundamental changes in the form of the basic equation, the solution of which is the final formula. That is why the proposed dependence cannot be used for the case of a hydraulic transport complex operating on water, i.e., when the concentration of the hydraulic mixture is zero. For these cases, it is necessary to obtain a new equation that reflects the relationship between the flow and pressure characteristics of the pumps and the pipeline, reduce it to a dimensionless form, and obtain the corresponding solution. This is the goal of further research by the authors.

**Conflict of interest**

Authors state no conflict of interest.

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## ОБЧИСЛЕННЯ ВИТРАТИ ГІДРОСУМІШІ ГІДРОТРАНСПОРТНОГО КОМПЛЕКСУ АНАЛІТИЧНИМ ШЛЯХОМ

*Семененко Є., Медведєва О., Рижова С., Коваль Н., Хамініч О., Жанакова Р.*

**Анотація.** В статті запропоновано мультиплікативну формулу для розрахунку в аналітичному вигляді витрати гідросуміші гідротранспортного комплексу з урахуванням параметрів трубопрововоду та відцентрових насосів, що використовуються, а також характеристик полідисперсного твердого матеріалу, що транспортується. Формула передбачає використання для опису залежності коефіцієнту гідравлічного опору від числа Рейнольдса степеневого закону замість логарифмічного та апроксимації витратно-напірної характеристики відцентрових насосів спеціальною степеневою функцією замість традиційного квадратичного двочлена. Мультиплікативність формули, що запропонована, полягає в тому, що величина витрати гідросуміші гідротранспортного комплексу визначається добутком трьох співмножників, одного розмірного та двох безрозмірних, кожен з яких враховує вплив окремої групи факторів. Характерестична витрата є єдиним розмірним співмножником та враховує вплив діаметру трубопроводу, кінематичного коефіцієнту в'язкості гідросуміші. Параметр полідисперсності матеріалу, що транспортується, є безрозмірною величиною, враховує вплив геодезичного ухилу магістралі та параметри дрібних фракцій, тобто частинок з крупністю від 0,1 мм до 2 мм, й шматкових фракцій, тобто частинок з крупністю більше за 2 мм. Параметр насосів, що встановлено, теж є безрозмірною величиною, та визначається концентрацією гідросуміші, властивостями твердої фази та її тонких фракцій, тобто частинок з крупністю менше за 0,1 мм, а також коефіцієнтами апроксимації витратно-напірних характеристик відцентрових насосів, що встановлено. Формула дозволяє досліджувати залежність витрати гідросуміші гідротранспортного комплексу від різних величин аналітичними методами, наприклад визначати екстремуми функції шляхом порівняння її похідних з нулем, або визначення припустимих інтервалів змінення параметрів, що входять до форомули, шляхом обмеження витрати гідросуміші.

**Ключові слова:** напірний гідротранспорт, витратно-напірна характеристика, відцентровий насос, гідросуміш, витрата